Int. J. Heat Mass Transfer. Vol. 2, pp. 185–198. Pergamon Press 1961. Printed in Great Britain.

SKIN FRICTION AND HEAT TRANSFER FOR INCOMPRESSIBLE LAMINAR FLOW OVER POROUS WEDGES WITH SUCTION AND VARIABLE WALL TEMPERATURE

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(*Received* 13 *June* 1960, *and in revised form* 15 *September* 1960)

Abstract—The boundary-layer differential equations for laminar flow over permeable wedges with suction, including isothermal and variable wall temperature distributions, have been solved. By use of the present solutions, the heat transfer and skin friction for laminar flow over any arbitrary geometry can be calculated. The application of the solutions to the binary gas flow and condensation is demonstrated, and the relation between the mass condensed, the heat removed, and the surface temperature is derived.

Résumé—Les équations différentielles de la couche limite laminaire avec aspiration sur des dièdres poreux, dans le cas de distributions de température de paroi variables ou non, ont été résolues. L'utilisation de ces solutions permet le calcul de la transmission de chaleur et du frottement à la paroi pour un écoulement laminaire sur une forme quelconque. Il est démontré que ces solutions sont applicables à l'écoulement d'un gaz binaire avec condensation, la relation entre la masse condensée, la chaleur enlevée et la température de surface est donnée.

Zusammenfassung—Die Grenzschichtgleichungen der Laminarströmung über durchlässige Kanten mit Absaugung wurden für konstante und veränderliche Verteilungen der Wandtemperatur gelöst. Mit Hilfe dieser Lösungen kann der Wärmeübergang und die Oberflächenreibung bei Laminarströmung für beliebig geformte Körper berechnet werden. Eine Anwendung der Lösungen auf den binären Gasstrom und Kondensation wird gegeben und die Beziehung zwischen Kondensatmenge, abgeführter Wärme und Oberflächentemperatur abgeleitet.

Аннотация—В статье приводятся решения дифференциальных уравнений пограничного слоя для ламинарного потока, обтекающего клин с проницаемыми поверхностями, через которые отсасывается пограничный слой. При этом, поверхность клина может быть как изотермической, так и неизотермической. Используя настоящие решения, можно вычислить теплоперенос и поверхностное трение для ламинарного потока по поверхности тела произвольной формы. Решение распространено на случаи потоков бинарного газа и конденсации. Получено соотношение между конденсированной массой, потоком тепла и температурой поверхности тела.

NOMENCLATURE

- A, constant;
- B, constant;
- C, constant;
- C_1, C_2 integration constants;
- C_{fx}, local friction coefficient $= \dfrac{[\mu(\partial u/\partial y)_w]}{\rho U^2/2}$
- c_p, specified heat at constant pressure;
- D, coefficient of ordinary diffusion;
- E, constant;
- f, dimensionless stream function equation (5), $f = \psi/\sqrt{(Ux\nu)}$;
- h, heat transfer coefficient $= q/(T_w - T_\infty)$;
- h_{fg}, latent heat of condensation;
- k, thermal conductivity of fluid
- $\dot{m}$, rate of condensation;
- m, Eulers number, equation (6);
- n, wall temperature parameter, equation (7);
- Nu_x, local Nusselt number $= hx/k$;
- p, heat flux parameter appearing in $q_x = Ex^p$;
- Pr, Prandtl number $= \mu c_p/k$;

N

q_x, local heat flux;
Re_x, Reynolds number $= Ux/\nu$;
Sc, Schmidt number $= \nu/D$;
T, temperature;
u, velocity component in x direction;
U, velocity component in x direction at the outer edge of the boundary layer;
v, velocity component in y direction;
W, mass fraction or concentration;
x, co-ordinate along the wall surface;
y, co-ordinate normal to the wall surface;
η, dimensionless distance from the wall, equation (5);
α, thermal diffusivity;
μ, dynamic viscosity;
ν, kinematic viscosity $= \mu/\rho$;
ρ, density;
θ, dimensionless temperature, equation (5);
ϕ, dimensionless concentration $= (W - W_w)/W_\infty - W_w)$;
ψ, stream function, equation (4).

Subscripts
w, wall ($y = 0$);
1, vapor component;
iso, isothermal wall;
∞, conditions at outer edge of boundary layer.

Superscripts
Primes (′) denotes differentiation;
——— three-dimensional variables.

1. INTRODUCTION

It is well known that the skin friction and heat transfer in incompressible laminar flow over wedge-shaped bodies can be accurately calculated by solving the boundary layer differential equations, provided that the prescribed surface temperature is a power function of the distance measured from the leading edge. For flow over an arbitrary body shape with known pressure or velocity distribution where there exists no similarity the skin friction and heat transfer are conventionally found by an approximate method, either the integral method or the equivalent wedge flow approximation. Both of these two methods yield sufficiently accurate results for most engineering applications. To apply the equivalent wedge flow method for the prediction of skin friction and heat transfer it is necessary to have the solutions of the boundary layer equations for wedge type flows. Such solutions are generally available for the impermeable wedge [1, 2, 3, 4, 5] and for the porous wedge with blowing [6, 7, 8, 9, 10, 11] including the consideration of a variable surface temperature distribution.

However, in the case of a porous wedge with suction [8, 9, 10] the available information is very limited. For the flat plate with suction Emmons and Leigh [10] have furnished solutions of the momentum equation, while the corresponding heat transfer results for the isothermal porous plate were presented by Schlichting and Bussemann [8] and Hartnett and Eckert [9]. These latter investigations [8, 9] also included heat transfer and skin friction results for an isothermal plane stagnation region with wall suction. This limited information is not sufficient to allow the application of the equivalent wedge flow method to flow over an arbitrary shaped body with wall suction. Such a flow situation may occur in boundary layer control applications, in flow over porous surface such as parachutes, etc.

It is the purpose of this paper to report the solutions of the boundary layer equations for laminar flow over permeable wedges with suction, including isothermal and variable wall temperature distributions. The flow is assumed to be incompressible with constant properties and the Prandtl number is taken to be 0·73.

2. DIFFERENTIAL EQUATIONS AND BOUNDARY CONDITIONS

The boundary-layer equations for two-dimensional steady incompressible laminar flow with constant properties are:

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

Momentum:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2} + \rho U \frac{dU}{dx} \qquad (2)$$

Energy:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad . \qquad (3)$$

Since the Mach number has been assumed to be small (i.e. incompressible flow) the dissipation and pressure gradient terms are negligible and have been omitted from the energy equation. The boundary conditions are:

Momentum:

$$u = 0 \quad \text{at } y = 0$$
$$v = v_w(x) \text{ at } y = 0$$
$$u = U(x) \text{ as } y \to \infty$$

Energy:

$$T = T_w(x) \text{ at } y = 0$$
$$T = T_\infty \quad \text{as } y \to \infty.$$

The continuity equation can be satisfied by introducing a stream function, ψ, such that

$$u = \frac{\partial \psi}{\partial y} \qquad v = -\frac{\partial \psi}{\partial x} \quad . \tag{4}$$

The momentum and energy equations can be transformed to the corresponding ordinary differential equations by the following substitutions [9]:

$$\left.\begin{aligned} \eta &= \frac{y}{x}\sqrt{\left(\frac{Ux}{\nu}\right)} \\ f &= \frac{\psi}{\sqrt{(Ux\,\nu)}} \\ \theta &= \frac{T - T_w(x)}{T_\infty - T_w(x)} \end{aligned}\right\} \tag{5}$$

where f and θ are assumed to be a function of η only.

For flow over an infinite wedge of angle $\beta\pi$ [$\beta = 2m/(m+1)$] the free stream velocity U, just outside the boundary layer, can be shown to be:

$$U = Ax^m \tag{6}$$

where m is known as the Euler number.

It will be assumed that the temperature difference between the wall and the free stream varies as Bx^n, i.e.:

$$T_w(x) - T_\infty = Bx^n. \tag{7}$$

By applying the above relations, the momentum and energy equations are transformed into the following forms:

Momentum:

$$f''' + \frac{m+1}{2} ff'' - m\,[(f')^2 - 1] = 0 \tag{8}$$

Energy:

$$\theta'' + Pr\frac{m+1}{2} f\theta' + nPr\,f'\,(1-\theta) = 0. \tag{9}$$

The velocity components, u and v, can be expressed in terms of the new variables as:

$$\left.\begin{aligned} \frac{u}{U} &= f' \\ \frac{v}{U} &= \frac{1}{2\sqrt{(Re_x)}}[-f(m+1) + f'\eta\,(1-m)] \end{aligned}\right\} \tag{10}$$

where $Re_x = Ux/\nu$.

The boundary conditions are:

$$\left.\begin{aligned} &\left.\begin{aligned} f' &= 0 \qquad (u = 0) \\ f &= f_w = -\frac{2}{m+1}\frac{v_w}{U}\sqrt{(Re_x)} \\ \theta &= 0 \end{aligned}\right\} \text{at } \eta = 0 \\ &\left.\begin{aligned} f' &= 1 \\ \theta &= 1 \end{aligned}\right\} \text{as } \eta \to \infty \end{aligned}\right\}. \tag{11}$$

In the second boundary condition the dimensionless suction parameter f_w may be taken as any constant value, thereby ensuring that the solution will depend only on the new variable η. This requires the suction velocity distribution to vary in the following manner:

$$v_w \sim \frac{m+1}{2} x^{(m-1)/2}.$$

Thus, for the flat plate $v_w \sim (1/\sqrt{x})$ and for the plane stagnation point, v_w = constant. For any other distribution of suction the solutions will depend on x and the resulting velocity profile will not be affine.

Note that the momentum equation is independent of the energy equation for constant fluid properties and hence can be solved independently. Once the solution of the momentum equation is obtained the $f(\eta)$ and $f'(\eta)$ values are utilized to solve the energy equation.

3. INTEGRATED FORMS OF THE BOUNDARY LAYER EQUATIONS AND THE METHOD OF SOLUTION

In the present analysis the momentum and energy equations were written in their integrated forms and then solved by iteration. The integrated form of the momentum and energy equations, together with their boundary conditions, are as follows:

$$f = f_w + \int_0^\eta f' \, d\eta \tag{12}$$

$$f' = \int_0^\eta f'' \, d\eta \tag{13}$$

$$f'' = \exp\left[-\int_0^\eta \frac{m+1}{2} f d\eta\right]\left\{m\int_0^\eta (f'^2 - 1) \times \exp\left[\int_0^\eta \frac{m+1}{2} f d\eta\right] d\eta + C_1\right\} \tag{14}$$

$$\left.\begin{aligned} \theta = -n\int_0^\eta \Big\{ & \exp\left[-\int_0^\eta Pr \frac{m+1}{2} f d\eta\right] \\ & \times \int_0^\eta Pr\, f'(1-\theta) \\ & \times \exp\left[\int_0^\eta Pr \frac{m+1}{2} f d\eta\right] d\eta \Big\} d\eta \\ & + C_2 \int_0^\eta \exp\left[-\int_0^\eta Pr \frac{m+1}{2} f d\eta\right] d\eta \end{aligned}\right\} \tag{15}$$

where

$$C_1 = \frac{1 - m\int_0^\infty \left(\exp\{-\int_0^\eta [(m+1)/2] f d\eta\} \int_0^\eta (f'^2 - 1)\exp\{\int_0^\eta [(m+1)/2] f d\eta\} d\eta\right) d\eta}{\int_0^\infty \exp\{-\int_0^\eta [(m+1)/2] f d\eta\} d\eta} \tag{16}$$

$$C_2 = \frac{1 + n\int_0^\infty \left(\exp\{-\int_0^\eta Pr\,[(m+1)/2] f d\eta\} \int_0^\eta Pr\, f'(1-\theta) \exp\{\int_0^\eta Pr\,[(m+1)/2] f d\eta\} d\eta\right) d\eta}{\int_0^\infty \exp\{-\int_0^\eta Pr\,[(m+1/)2] f d\eta\} d\eta} \tag{17}$$

The above equations have been solved by use of the E.R.A. 1103 computer. For any given Euler number, m, and suction parameter, f_w, an initial velocity profile $f_0'(\eta)$ was estimated and substituted into equation (12) to get an estimated $f_0(\eta)$. These $f_0'(\eta)$ and $f_0(\eta)$ were then used in equations (16), (14) and (13) consecutively to obtain the velocity profile, say $f_1'(\eta)$. This $f_1'(\eta)$ was usually different from $f_0'(\eta)$. In this case an intermediate value between $f_0'(\eta)$ and $f_1'(\eta)$ was used in place of $f_0'(\eta)$ and the above processes were repeated until a consistent value of $f'(\eta)$ was obtained.

In the case of the energy equation the temperature profile, say $\theta_0(\eta)$, was first computed directly from equations (17) and (15) consecutively for $n = 0$. For a given temperature parameter n this $\theta_0(\eta)$ was substituted into the right-hand sides of equations (17) and (15) to get an approximate temperature profile $\theta_1(\eta)$. An intermediate value between $\theta_0(\eta)$ and $\theta_1(\eta)$ was then used to obtain a better approximation for $\theta(\eta)$. These procedures were repeated to get the final result.

4. FRICTION COEFFICIENT AND HEAT TRANSFER

The friction coefficient C_{fx} is defined as:

$$C_{fx} = \frac{\tau_w}{\rho U^2/2} = \frac{\mu(\partial u/\partial y)_w}{\rho U^2/2}.$$

It is related to the integration constant C_1 by the following equation:

$$\frac{C_{fx}\sqrt{(Re_x)}}{2} = f_w'' = C_1. \tag{18}$$

The dimensionless heat transfer parameter, (i.e. the Nusselt number) is:

$$Nu_x = \frac{hx}{k} = \theta_w' \sqrt{(Re_x)}.$$

or

$$\frac{Nu_x}{\sqrt{(Re_x)}} = \theta_w' = C_2. \tag{19}$$

5. RESULTS AND DISCUSSIONS

The boundary-layer equations have been solved for several wedge angle and suction parameters. The temperature parameter n was varied from $-0{\cdot}5$ to 10 except for the plane stagnation flow where results for $n = -1$ are also included. The value of Prandtl number is taken as 0·73 (air).

The velocity profile and friction coefficient for flow over wedges with various suction are shown

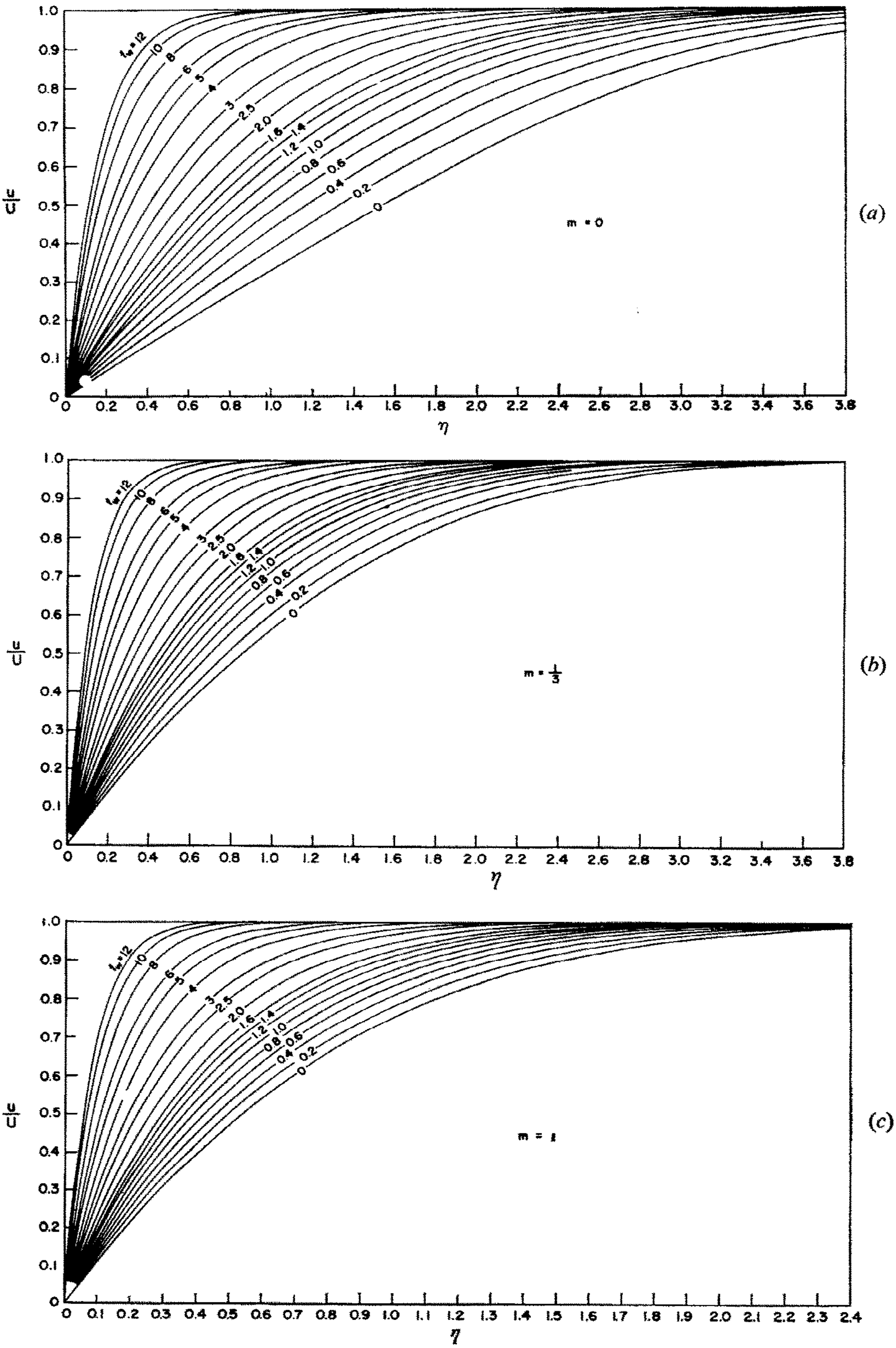

FIG. 1. Velocity profile for flow over wedges with various suction. (a) $m = 0$; (b) $m = \frac{1}{3}$; (c) $m = 1$.

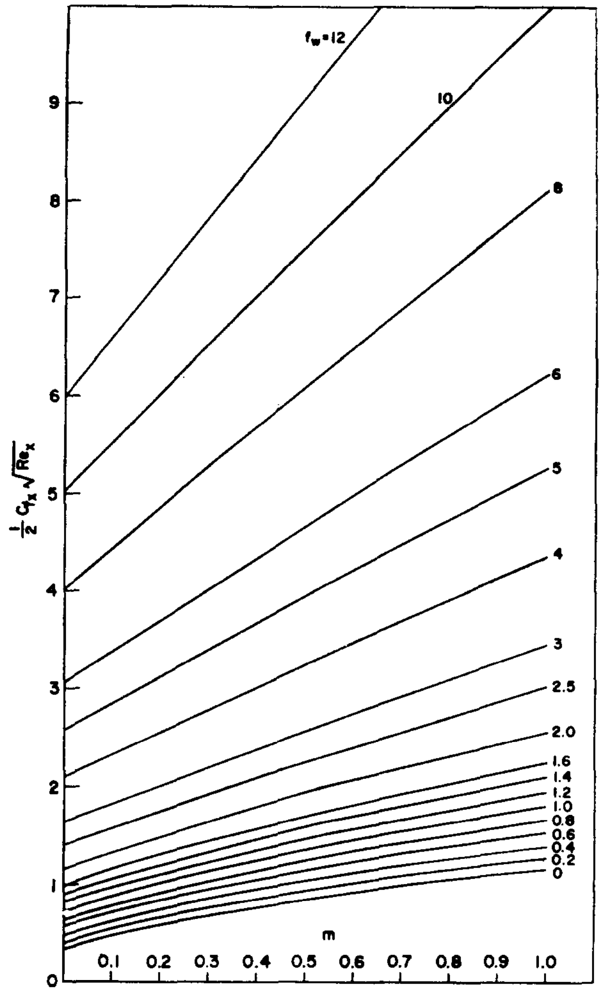

FIG. 2. Friction coefficient (flow over wedges with various suction).

on Figs. 1 and 2, respectively. As expected Fig. 1 shows that the velocity boundary-layer thickness decreases as the suction increases. This is responsible for the increase of skin friction as is revealed by Fig. 2. For large suction, Fig. 2 also shows that the friction coefficient is approximately a linear function of m.

The boundary-layer temperature profiles on a flat plate with power-function wall temperature variation are shown on Fig. 3 for eight values of the wall temperature exponent n, with Fig. 3(a) corresponding to the impermeable wall, while Fig. 3(b) and 3(c) are for the porous plate for values of the suction parameter equal to $f_w = 1{\cdot}0$ and $8{\cdot}0$, respectively. Similar presentations are given on Figs. 4 and 5 for two other wedge flows corresponding to Euler numbers of $\frac{1}{3}$ and 1.

It is seen from these figures that for a fixed wedge angle and fixed suction the thermal boundary-layer thickness is reduced with increase of n. Consequently, it is expected that the heat transfer would increase with increase of n. This conclusion is clearly verified by Figs. 6, 7 and 8 which present the dimensionless heat transfer, $Nu_x/\sqrt{(Re_x)}$, for the three wedge flows. These figures also show that the heat transfer is approximately a linear function of n for large values of suction.

Figure 9 reveals that for negative wall temperature gradient (negative n), the heat transfer for variable wall temperature is less than that for isothermal wall, and for positive wall temperature gradient (positive n) the heat transfer for the non-isothermal wall is always larger than that for isothermal wall. The ratio of non-isothermal wall heat transfer to the isothermal wall heat transfer is very sensitive to n for solid wall and small suction. However, this ratio increases very gradually with n for large suction. The same figure also shows that for a fixed suction and wall temperature variation, this ratio decreases with increase of free stream velocity gradient, or Euler number.

6. THREE-DIMENSIONAL STAGNATION FLOW

By means of Mangler's transformation [12] the three-dimensional axially symmetric flow can be transformed to the corresponding two-dimensional case. For three-dimensional stagnation flow the equivalent two-dimensional case would be the flow over a wedge with Euler number of $\frac{1}{3}$. It can easily be shown that the suction velocity, the temperature parameter, the friction coefficient and the Nusselt number for the three-dimensional stagnation flow are related to the associated two-dimensional case by the following equations:

Suction parameter:

$$\frac{\bar{v}_w}{\bar{U}}\sqrt{\bar{Re}_x} = -\frac{2}{\sqrt{3}} f_w. \qquad (20)$$

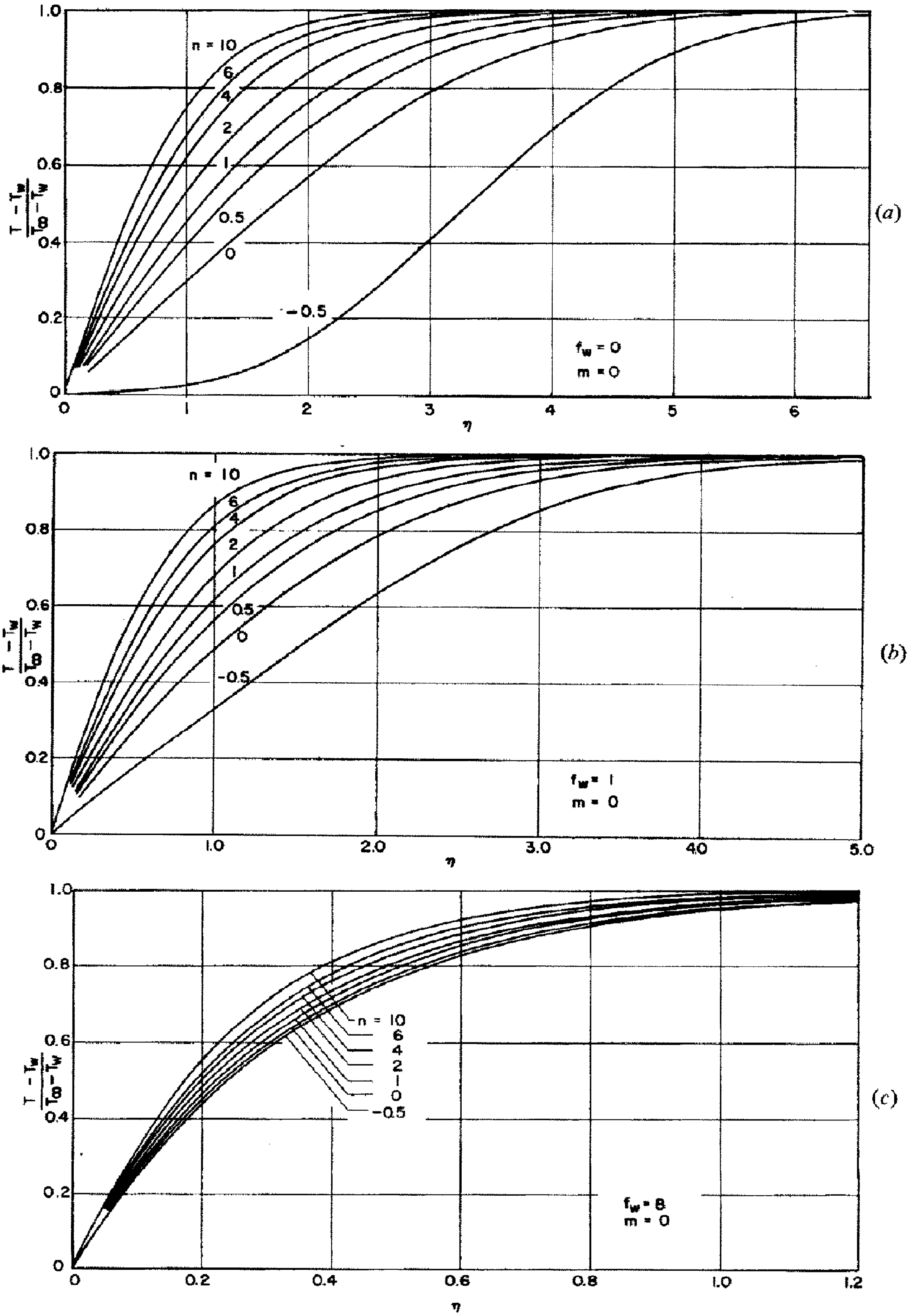

FIG. 3. Temperature profile for flow over a flat plate with various suction and variable wall temperature. (a) $f_w = 0$, $m = 0$; (b) $f_w = 1$, $m = 0$; (c) $f_w = 8$, $m = 0$.

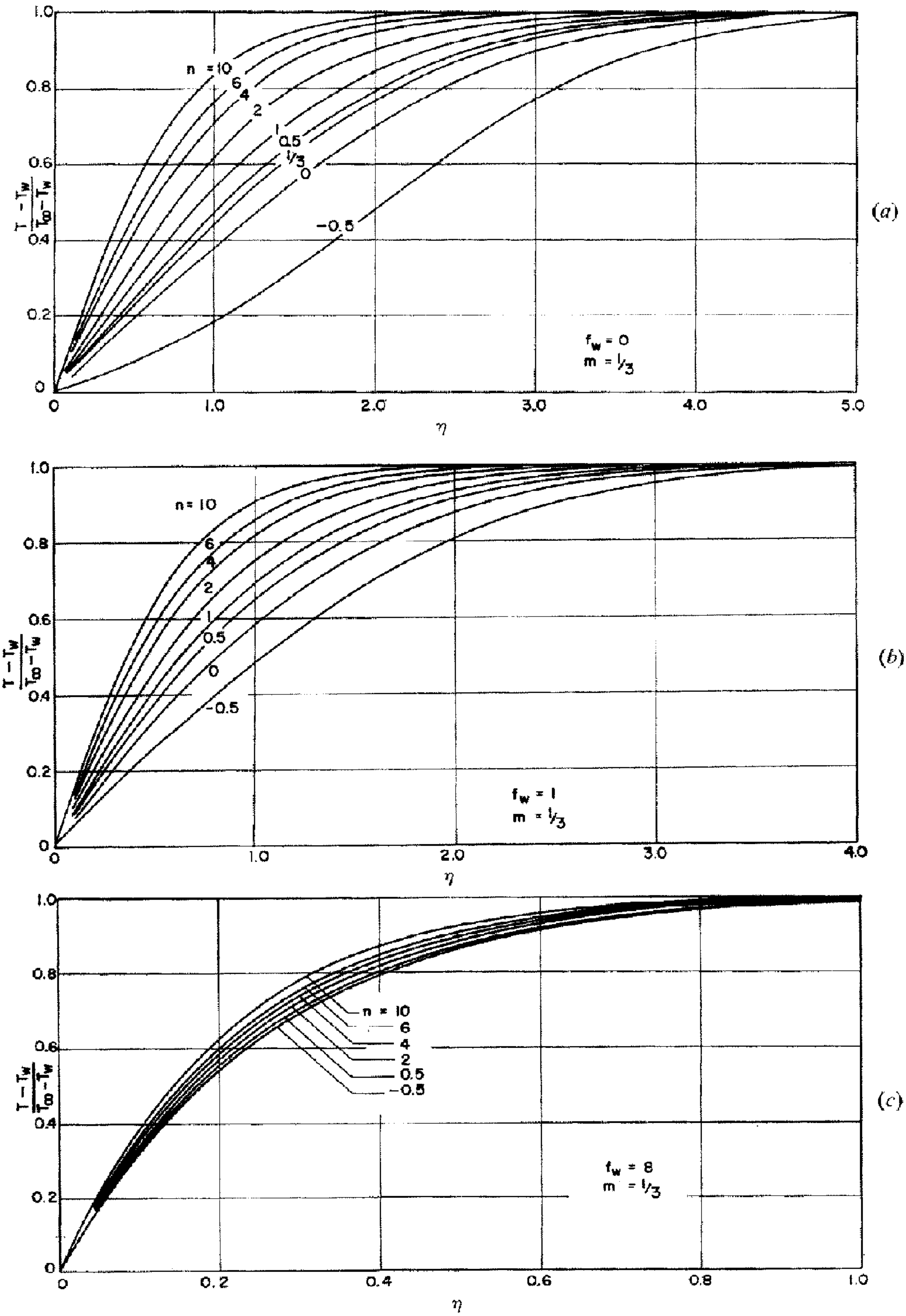

FIG. 4. Temperature profile for flow over wedge with various suction and variable wall temperature. (a) $f_w = 0$, $m = \frac{1}{3}$; (b) $f_w = 1$, $m = \frac{1}{3}$; (c) $f_w = 8$, $m = \frac{1}{3}$.

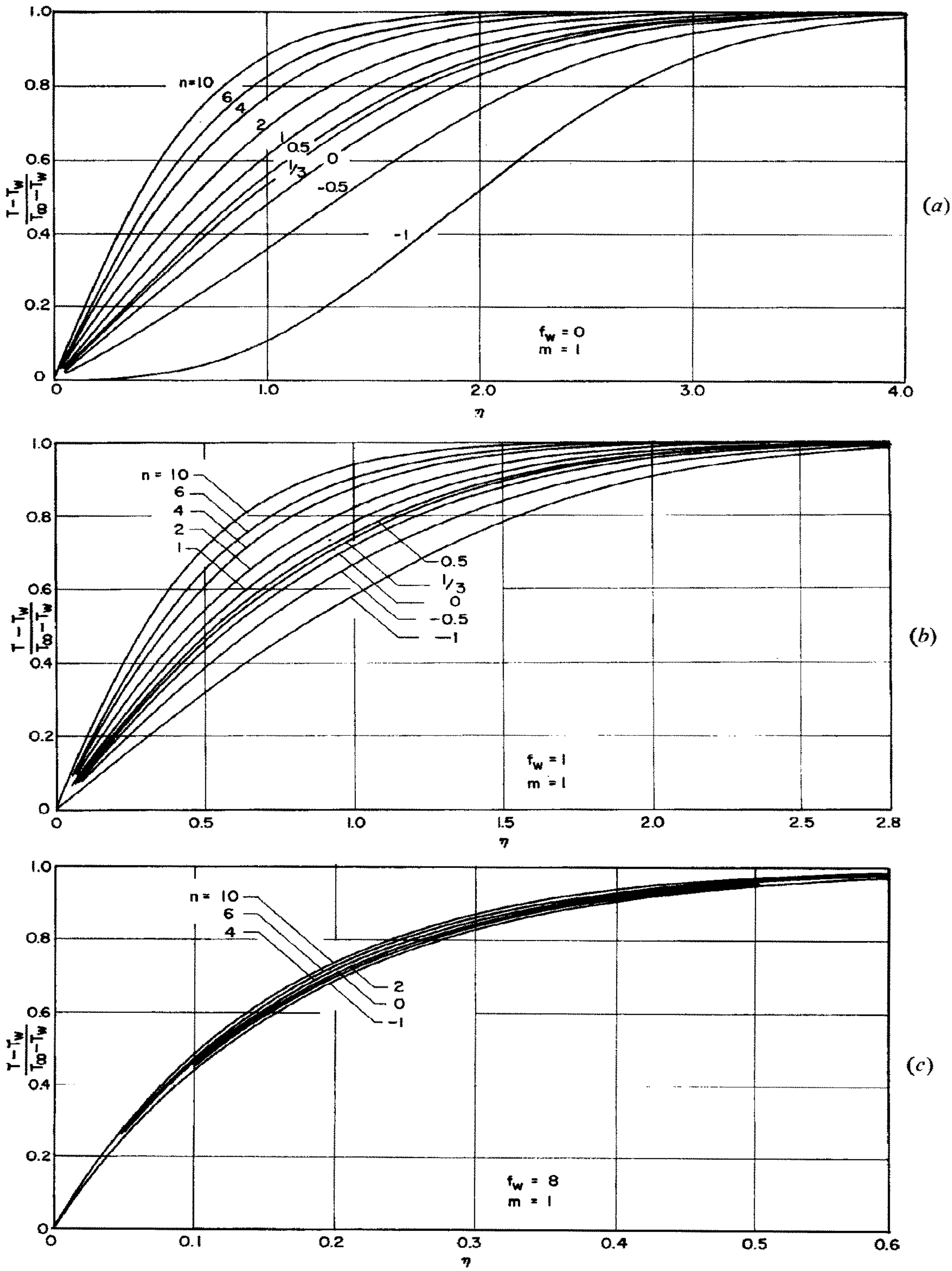

FIG. 5. Temperature profile for plane stagnation flow with various suction and variable wall temperature. (a) $f_w = 0$, $m = 1$; (b) $f_w = 1$, $m = 1$; (c) $f_w = 8$, $m = 1$.

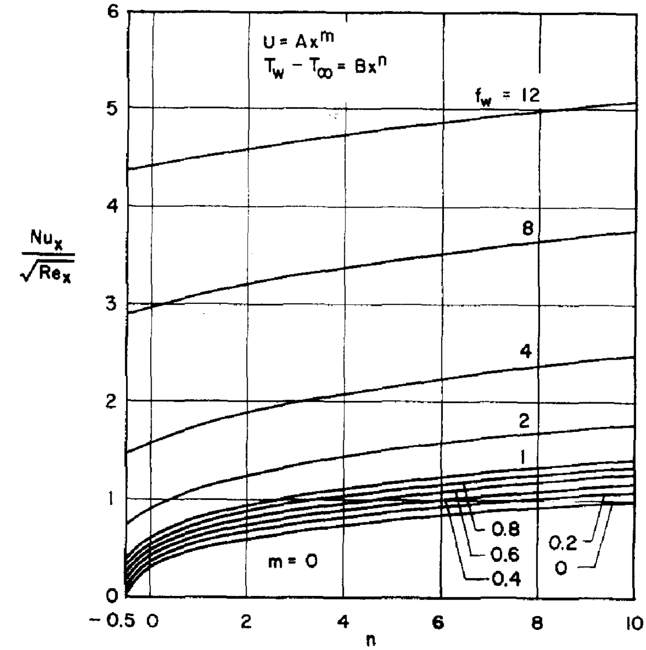

FIG. 6. Heat transfer for flow over flat plate with various suction and variable wall temperature.

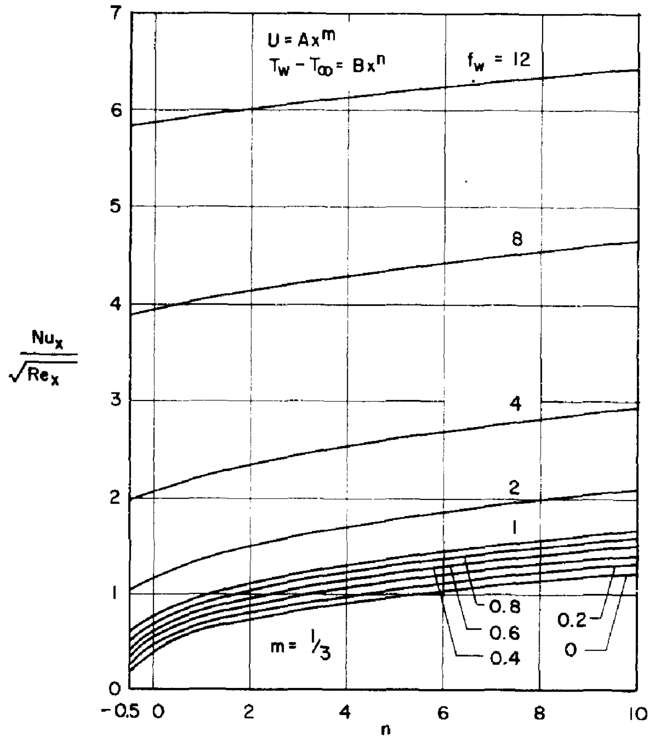

FIG. 7. Heat transfer for flow over wedge with various suction and variable wall temperature.

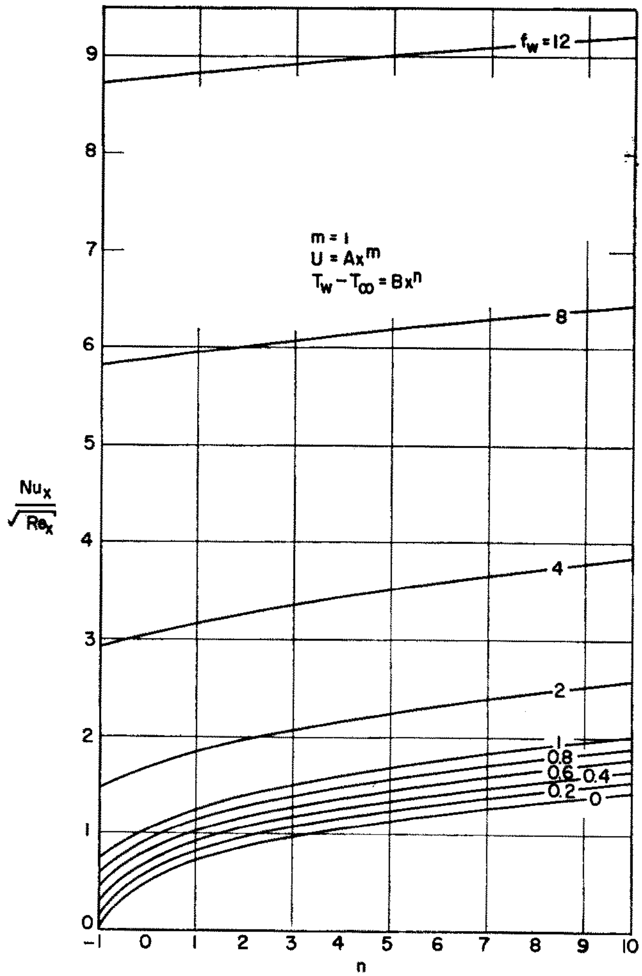

FIG. 8. Heat transfer for two-dimensional stagnation flow with various suction and variable wall temperature.

Temperature parameter:

$$\bar{n} = \frac{n}{3}. \tag{21}$$

Friction coefficient:

$$\overline{C}_{fx}\sqrt{\overline{Re}_x} = \sqrt{3}\, C_{fx}\sqrt{Re_x}\,\Big|_{m=1/3}. \tag{22}$$

Nusselt number:

$$\frac{\overline{Nu}_x}{\sqrt{\overline{Re}_x}} = \sqrt{3}\,\frac{Nu_x}{\sqrt{Re_x}}\Bigg|_{m=1/3} \tag{23}$$

where the superscript bar denotes the three-dimensional stagnation flow case.

Hence, in three-dimensional stagnation flow, if the suction velocity, free stream conditions and the wall temperature variation $\bar{n}(\overline{T}_w - \overline{T}_\infty = c\bar{x}^{\bar{n}})$ are given the f_w and n can be computed from equations (20) and (21) and the friction coefficient and Nusselt number can be found from equations (22) and (23), respectively.

7. APPLICATION OF THE WEDGE FLOW SOLUTION TO PREDICT THE SKIN FRICTION AND HEAT TRANSFER FOR LAMINAR FLOW OVER ARBITRARY BODY

For flow over any arbitrary two-dimensional body with given free stream velocity and wall

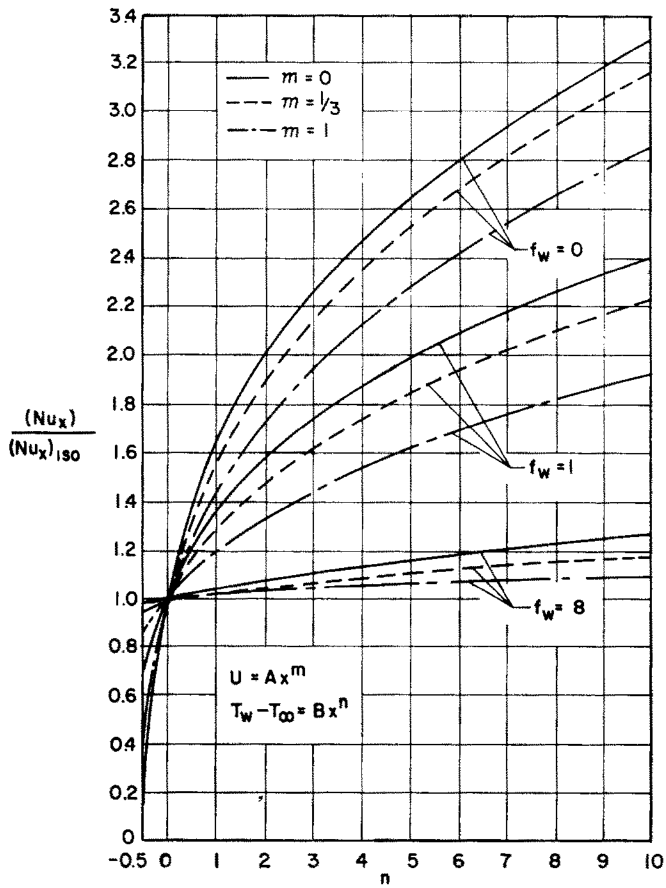

FIG. 9. Effect of wall temperature variation on heat transfer for flow over wedges with various suction.

temperature distribution the skin friction and heat transfer can be approximately calculated by use of the wedge flow solutions. First, the equivalent Euler number m and the approximate temperature parameter n can be found by the following equations:

$$m = \frac{x}{U}\frac{dU}{dx} \tag{24}$$

$$n = \frac{x}{T_w - T_\infty}\frac{dT_w}{dx}. \tag{25}$$

In these equations, x is measured from the leading edge or the forward stagnation point.

With m computed and f_w specified, the skin friction can be directly read from Fig. 2. The heat transfer can be found from Figs. 6, 7 and 8 after the temperature parameter n has been found from equation (25).

If the heat transfer distribution is prescribed as $q_x = Ex^p$, the wall temperature T_w can be determined from the heat transfer equation:

$$q_x = -k\left(\frac{\partial T}{\partial y}\right)_w = k\sqrt{\left(\frac{U}{\nu x}\right)}(T_w - T_\infty)\,\theta'_w \tag{26}$$

where θ'_w can be found from Figs. 6, 7 and 8 for a given m and f_w with $n = p + (1 - m)/2$.

8. APPLICATION OF WEDGE FLOW SOLUTION TO BINARY GAS FLOW AND CONDENSATION

In a binary gas flow, the concentration of the gaseous component can be found by solving the diffusion differential equation. For constant fluid properties and a Lewis number of unity the differential equations and boundary conditions for both the dimensionless concentration

$$\phi = \frac{W - W_w}{W_\infty - W_w}$$

and temperature

$$\theta = \frac{T - T_w}{T_\infty - T_w}$$

are identical [9]. Consequently, the concentration information can be immediately obtained from the energy equation solutions, provided that certain compatibility conditions are satisfied. Under the assumptions that the departure from thermodynamic equilibrium is negligible and that the wall surface remains wetted, the following conditions must be satisfied:

(a) The partial pressure of the condensing vapor must be equal to the vapor pressure of the liquid at the surface temperature. Therefore, a specification of the wall temperature fixes the partial pressure and, consequently, fixes the mass fraction of the condensing vapor at the wall. It is obvious that the mass fraction of the vapor must be greater in the free stream than at the wall surface if condensation is to occur. This requires the surface temperature to be lower than the free stream temperature.

(b) If there is no net flow of boundary-layer air into the wall surface this requires the convective velocity toward the wall to be exactly balanced by the diffusive velocity of the air away from the wall. In terms of dimensionless variables:

$$\frac{1 - W_{1w}}{W_{1w} - W_{1\infty}} = - \frac{(\partial\phi/\partial\eta)_w}{Sc\{[(m+1)/2]f_w\}}. \quad (27)$$

Therefore, the condensation rate f_w is related to the mass fraction of the condensing vapor W_{1w}.

(c) The heat transferred to the wall by convection and by the condensing vapor must be removed from the wall by a suitably distributed heat sink:

$$q_x = h_x(T_\infty - T_w) + \dot{m}h_{fg}.$$

Rearranging and expressing in terms of dimensionless variables this may be stated:

$$\frac{q_x}{k(T_\infty - T_w)} \sqrt{\left(\frac{\nu x}{U}\right)} =$$

$$= \theta'(0) + \frac{m+1}{2} f_w \, Pr \, \frac{h_{fg}}{c_p(T_\infty - T_w)}. \quad (28)$$

For example, if we restrict our attention to the flat plate geometry ($m = 0$) with given free stream conditions, the selection of a constant wall temperature T_w (where $T_w < T_\infty$) fixes the

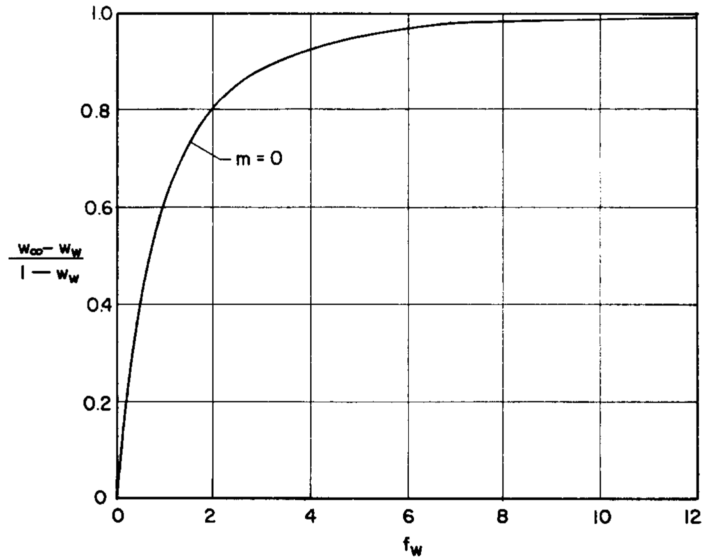

FIG. 10. Effect of suction on concentration (no net air flow at the wall).

wall mass fraction at a constant value. For the assumed constant wall temperature boundary conditions there is a unique relationship between $\phi'(0)$ [identical in value to $\theta'(0)$] and f_w as shown in Fig. 6, allowing the wall mass fraction to be directly related to f_w through equation (27). This is shown on Fig. 10.

Finally, equation (28) must be satisfied and it is apparent that the heat sink q_x must vary as $1/\sqrt{x}$ to satisfy the compatibility relations. By this method it is a simple matter to prepare a table of values of the dimensionless heat removal requirements, the mass condensed and the resulting surface temperatures.

For flows over other wedge-type geometries ($m \neq 0$), the application of the reported results to the condensation problem becomes more complex. In such cases, the assumption of a constant wall temperature results in a constant value of the partial pressure at the wall. However, the free stream pressure is varying along the surface and consequently, the wall mass fraction will vary along the surface for a constant wall temperature condition. If the present solutions are to apply under such circumstances, the mass fraction term $W_{1\infty} - W_{1w}$ must vary as a power function x^n. For some wedge angles this condition may be satisfied. Alternatively, for a fixed wedge angle it may be possible to find a power function wall temperature distribution which will give rise to a wall mass fraction value which will satisfy the compatibility requirements imposed by equations (27) and (28). Detailed consideration of this case is beyond the scope of the present paper.

ACKNOWLEDGEMENTS

The financial support of the Numerical Analysis Center of the University of Minnesota is gratefully acknowledged. The help of Mr. W. Minkowycz in computer programming is appreciated.

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